**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with ** = 45 minutes and ** = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans:Let X be the random variable

X=60-10=50

X =50

z = X-u/sigma=50-45\8=0.625

P(z)=0.73241-0.7324=

**0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean ** = 38 and Standard deviation ** =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans:Let X be the age of employee.

X=44

Now z = X-u/sigma

z=44-38/6

z=1

Now P(z)=0.8413

For age 44 not going to happen is

1-0.8413=0.1587

True

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: X=30

z=X-u/sigma

z=30-38/6

z=-1.33

Now P(z)=0.1515

0.0918

Now multiplying the probability of under 30 age workers

400\*0.0918 = 36.72

**True**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans : As we know that if *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are two independent random variables then X1 + X2  N( µ1 + µ2 , 21 + 22 ) and X1 – X2   N( ( µ1 - µ2 , 21 -22)**

**Similarly if Z = aX + bY , where X and Y are as defined above i.e. Z is linear combination of X and Y, the Z  N( aµ1 + bµ2 , a2221 + b2221)**

**Therefore in this question**

**2X1  N(2µ , 42) and**

**X1 + X2  N( µ + µ ,  + ) = N( 2µ , 22)**

**2X1 – (X1 + X2)  N (4µ , 62)**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans:D. Let X be the age of employee.

P of getting a and b 0.99

Not getting = 1-0.99 = 0.01

As the value both side is same for a and b

0.001/2=0.005

Both side = -0.005 and 0.005

Z(-0.005)=2.58 Z(0.005)=-2.58

z = X-u/sigma

X =z\*sigma + u

a = -2.58\*20+100 = 48.5

**ANS D**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?